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Split Quaternions and Hyperbolic Spinor Representation of Transformations

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Abstract

In this study, a historical continuum of development of spinor theory is summarized. Although the name of spinor was first used by physicists, its' mathematical form was firstly introduced by Cartan in 1913. From past to present the literature on spinor becomes substantially extensive since it has applications to electron spin, quantum mechanics, electronic magnetic field and electric transmission lines.

After revisiting the geometrical and analytical description of a spinor, the equations for the rotation of spinors are given. The spinors have an essential role in numerous scientific area instead of the vectors. In this regard, one of the aim of this study is to give the hyperbolic spinor representations of Frenet, Bishop and Darboux equations for a non-null curve in $\mathbb{R}_1^3$, since these equations have vital importance in differential geometry.

Moreover, a different perspective is developed for the relationship between the rotations in $\mathbb{R}_1^3$ and their corresponding split quaternions in $\mathbb{E}_2^4$ making use of hyperbolic spinors. As a consequence, this treatment provides a natural extension of split quaternions and allows us to compare well known concepts of Lorentzian Kinematics with newly introduced.

Keywords: Spinor; Hyperbolic spinor; Split quaternion.

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