MODERN TOPICS IN THE GEOMETRY OF EINSTEIN SPACES

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Given a compact C^∞-differentiable manifold M, $\dim M = n$, the following question arises (René Thom, Strasbourg Math. Library, 1958, see [1]):

Are there any best (or nicest, or distinguished) Riemannian structures on M ?

A good candidate for such a privileged metric on a given manifold is an *Einstein* metric, if one considers the *best metrics* those of constant sectional curvature. More precisely, if the dimension of the manifold is greater than 2, a good generalization of the concept of constant sectional curvature might be the notion of *constant Ricci curvature* [1].

A Riemannian manifold (M, g) of dimension $n \geq 3$ is called an *Einstein space* if $Ric = \lambda \cdot id$, where trivially $\lambda = \kappa$, with k the (normalized) *scalar curvature*; in this case one easily proves that $\lambda = \kappa = constant$.

We recall the fact that any 2-dimensional Riemannian manifold satisfies the relation $Ric = \lambda \cdot id$, but the function $\lambda = \kappa$ is not necessarily a constant. It is well known that any 3-dimensional Einstein space is of constant sectional curvature. Thus the interest in Einstein spaces starts with dimension $n = 4$.

Singer and Thorpe [4] discovered a symmetry of sectional curvatures which characterizes 4-dimensional Einstein spaces. Later, this result was generalized by B.Y. Chen e.a., in [2], to Einstein spaces of even dimensions $n = 2k \geq 4$. We established in [3] curvature symmetries for Einstein spaces of arbitrary dimension $n \geq 4$.

References

[1] A. Besse, *Einstein Manifolds*, Springer, Berlin, 1987.

[2] B.Y. Chen, F. Dillen, L. Verstraelen, L. Vrancken, *Characterizations of Riemannian space forms, Einstein spaces and conformally flat spaces*, Proc. Amer. Math. Soc. **128** (2000), 589-598.

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[4] I.M. Singer, J.A. Thorpe, *The curvature of 4-dimensional Einstein spaces*, Global Analysis (Papers in Honor of K. Kodaira), Univ. Tokyo Press, Tokyo, 1969, 355-365.